Estimation of turbulence strength directly from target images

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Abstract: When collecting images through turbulence it is always useful to have an estimate of turbulence strength during the time of the observations. We show how Fried’s parameter can be estimated from images of extended objects in the case of vertical-, and horizontal-path imaging scenarios.

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1. Introduction

Measurement of optical effects of atmospheric turbulence serves several purposes

1) basic meteorological research,
2) performance prediction of astronomical telescopes, long-range surveillance cameras, free-space laser communications systems, directed-energy weapons, and of the associated adaptive optics (AO) systems,
3) provision of point-spread function (PSF) information for subsequent de-blurring, when executed simultaneously or almost-simultaneously with the target observations.

to mention only a few applications. In this communication we focus on the last point.

Image restoration methods such as speckle imaging [1,2], Richardson-Lucy deconvolution [3,4], Wiener-filter deconvolution [5] and others rely on the availability of a PSF, or in general, a transfer function. This transfer function combines the effects of: diffraction by the sensor’s aperture, turbulence, and static aberrations of the imaging system.

The turbulent PSF is completely specified by known optical parameters, such as observing wavelength and the aperture of the sensor, and one unknown describing the integrated effect of the atmospheric turbulence between the source and the observer. This unknown is usually the Fried parameter \( r_0 \) [6]. Alternatively, one can express the PSF using the integrated value of \( C_n^2 \) along the propagation distance.

In night-time astronomy usually an unresolved reference star is observed simultaneously with, or after the target to naturally provide an estimate of the PSF. Solar observations unfortunately do not provide reference point sources. Same is true for surveillance applications. In ground-to-ground scenarios, methods based on image motion are most commonly used [7] but their accuracy is limited by non-atmospheric sources of image jitter (such as platform vibration, movement). In this paper we propose a different approach, following the “spectral ratio” idea [8].

2. Motivation

Object-cancelling transformation, as a way of removing object being observed from the image formation equation, was first proposed by von der Lühe [8]. Here, we propose an original transformation, which we call “Fourier contrast” method.

Image formation equation, expressed in the Fourier domain, is:

\[
I(\vec{u}) = O(\vec{u})H(\vec{u})
\]  

(1)

where \( \vec{u} \) is a spatial frequency vector in the Fourier plane and \( I(\vec{u}) \), \( O(\vec{u}) \) and \( H(\vec{u}) \) stand for Fourier transforms of the (instantaneous) image, object, and speckle PSF, respectively. For each frequency we calculate the mean values and standard deviations of the power spectra:

\[
C_f(\vec{u}) \equiv \frac{\text{var}(\|H(\vec{u})\|^2)^{1/2}}{\langle |H(\vec{u})|^2 \rangle} = \frac{\text{var}(\|O(\vec{u})|^2)^{1/2}}{\langle |O(\vec{u})|^2 \rangle} \cdot \frac{\text{var}(\|H(\vec{u})|^2)^{1/2}}{\langle |H(\vec{u})|^2 \rangle} = \var(C_f(\vec{u}))
\]  

(2)

where \( \langle \rangle \) denotes average and \( \text{var}(\cdot) \) denotes variance. We use the term “contrast”, and denote it with letter \( C \) as is customary in research pertaining to speckle [9]. The short-exposure optical transfer function (OTF), \( H_f(\vec{u}) \), is:
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Y -
fscintillation can -

- say -

rrier contrasts from observations -

- and -

z is the distance from the exit pupil to the image plane. It can be seen that the instantaneous OTF is the amplitude random walk composed of N equal-length, randomly phased phasors. Of interest to us are the first and the second central moments of $|H_i(x_i)|^2$. They are the same, except for the normalization constant, as the moments of intensity. Details of these derivations can be found in earlier papers [10,11]. Here we present the final result based on the partially-developed speckle model:

$$
\left\langle |H_i(x_i)|^2 \right\rangle = N(u) + N_1(u) M(u,1)
$$

$$\text{var} \left( |H_i(x_i)|^2 \right) = N(u) + 2N_1(u) + 4(N_2(u) + N_1(u))^2 + N_1(u)^2 + N_1(u)^2 M(u,2) - \left\langle |H_i(x_i)|^2 \right\rangle^2
\tag{4}
$$

where $M(.)$ is the characteristic function of the phase differences and $N_n = N!/(N-n-1)!$. This characteristic function is related to $r_0$ through the “short-exposure” structure function [6]. Also $N$, the number of OTF cells in the area of overlap at a particular spatial frequency, is related to $r_0$ [11]. The “Fourier contrast” method consists of fitting pre-computed analytic templates – Equation 4 – corresponding to, say 100 $r_0$’s in the expected variability range to empirically obtained Fourier contrasts from observations – Equation 2. This takes 0.02sec on a desktop PC (CPU @ 3.4GHz, 4 cores, 8GB of RAM). The value of $r_0$ with the smallest mean squared error is selected as the most probable one.

Fig. 1. Left: azimuthally-averaged simulated Fourier contrasts (solid lines) and predictions of the partially-developed speckle model (dashed lines). Size of the telescope aperture was set to 1.5m and 512x512 arrays were used for phase screens. The wavelength was 500nm and this is also the wavelength at which the Fried parameter values are quoted. For each level of turbulence strength, $r_0 = 2, 6$, or 10cm at 500nm, one hundred PSFs were generated. No noise was added to the images. Right: Accuracy of the new approach compared to the “spectral ratio” method of von der Lühe [8], based on images of artificial satellites. Noise-less simulation. Both figures taken from [11].

3. Vertical-path imaging

In applications such as astronomy or space situational awareness with large telescopes the effect of scintillation can be neglected due to the aperture-averaging effect. Here, the theory works very well. Figure 1 shows the azimuthally-averaged Fourier contrast – Equation 2 – computed from the simulated speckle images together with the predictions of the proposed model (dashed curves). It can be seen that the agreement is good.
4. Horizontal-path imaging

In horizontal-, and slant-path imaging through “deep” turbulence scintillation starts to play a role. The variance of intensity and OTF is therefore increased compared to the case of pure phase variations. Currently, the model does not account for variation of amplitudes of the OTF cells. Still, at least empirically, we are able to quantify r_0's variability from Fourier contrast even in the case of non-negligible scintillation.

This is illustrated by an example. Experiments were carried out at Fraunhofer IOSB in Ettlingen over a 2.5 km experimental path on November 7th 2012. The experimental path covers urban area of downtown Ettlingen, a few fields and a small portion of the forest. The light sources were implemented as follows: Holes of diameters ranging from 1 cm (quasi point-source) to 5 cm (isokinetic patch) were cut in opaque masks and these masks were placed, sequentially, in front of a high-power lamp. The receivers are located inside the Adaptive Optics laboratory at a height of approximately 20m from the ground. Light from the source was collected with an 18 cm telescope and directed to the imaging camera running at 30 fps. Exposure times were ranging from 4 to 17 ms. Turbulence strength, C_n^2 estimated from image motion, was around 10^{-14} m^{-2/3} throughout the day.

In Figure 2 we show how Fourier contrast can discriminate between good and bad seeing conditions. For observations shown in the leftmost panel, where the mean PSF was relatively narrow, the “dark hole” in the Fourier contrast map is correspondingly elongated showing increased wavefront coherence. For observations characterized by stronger turbulence, the mean PSF was broader and the Fourier contrast increased faster. This is also illustrated in azimuthal averages in the right panel of Figure 2.

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5. References
